Generalized and Average Post Detection Integration Methods for Code Acquisition

Giovanni Emanuele Corazza and Raffaella Pedone

DEIS/ARCES
University of Bologna, Viale Risorgimento 2
40136 Bologna, Italy
Tel. +39 051 2093054, Fax +39 051 2093053
e-mail: {gecorazza, rpedone}@deis.unibo.it

***Abstract*– In the context of spread spectrum systems, this paper focuses on Post Detection Integration (PDI) for code acquisition in the presence of a frequency error. The PDI scheme is obtained through the theoretical likelihood ratio test (LRT) computation, and two approaches are considered: the generalized LRT, in the form of the special case STML-PDI (Single Term Maximum Likelihood-PDI), and the average LRT (A^2LLRT). The methods are developed in the AWGN channel but are applicable to fading channels, too. Possible applications are in cellular CDMA networks and in satellite navigation and positioning systems.**

I. Introduction

In the last years, the interest towards Spread Spectrum (SS) techniques has been widely increased, including new horizons such as CDMA (Code Division Multiple Access) for mobile communications in the already assessed range of applications, mainly represented by military communications, and navigation/positioning/ranging. The largely acknowledged robustness against intentional interference, interception and multipath fading, the potential accurate delay measurements, and the new code division multiple access method are the most relevant supporters of SS techniques diffusion. A very recent extension to the applicability of CDMA is represented by the foreseen use of the 3G W-CDMA (Wideband-CDMA) radio interface in future S-DMB (Satellite-Digital Multimedia Broadcasting) satellite networks, such as the one addressed in the IST-project MAESTRO [1]. The recent vast request of multimedia services for mobile users has in fact strongly reinforced the topic of integration between the terrestrial and satellite networks, aiming at the design of hybrid inter-working architectures.

A critical aspect of SS systems is represented by code synchronization, which is fundamental for correct reception. This paper focuses in particular on the first synchronization phase, i.e. code acquisition, that provides the coarse code epoch estimation that will be refined by code tracking. In this context, the signal-to-noise ratio (SNR) is usually so low that no accurate parameter estimation is typically feasible before code synchronization. Besides phase uncertainty, also the presence of a frequency error, due to the oscillators mismatch and the transmitter/receiver possible motions, must be considered. To cope with the degradation induced by the frequency offset, code acquisition usually adopts a windowing technique [2], limiting coherent integration to a sub-section of the transmission pseudonoise (PN) sequence. The residual integration is performed after detection and is referred to as Post Detection Integration (PDI). PDI improves the SNR notwithstanding the noise-signal cross terms introduced by possible non linearities.

In the literature, many different PDI schemes have been proposed, among which the classic Non Coherent PDI (NC-PDI) [2], [3] and the Differential PDI (D-PDI) [4]. Differential integration can also be used for first step detection, as in [5].

This paper illustrates the results of the application of a theoretical procedure to optimize PDI performance, based on two different approaches to solve the frequency error uncertainty. The first strategy is based on the application of the Maximum Likelihood criterion applied through the generalized likelihood ratio test (LRT) [6], [7]. The second novel method eliminates the uncertainty through the appropriate statistical modelling of the unknown frequency offset, and computing the average LRT. The two resulting solutions are compared to find out possible performance improvements. The methods proposed in the literature are considered as a benchmark.

II. System Model

The system model is the typical pilot-aided timing acquisition scheme [2]. The received signal under hypothesis H_1 (pilot present) and under hypothesis H_0 (no pilot, neglecting self-noise) can be written as:

$$r(t) = \begin{cases} H_1: & p(t) + n(t) \\ \\ H_0: & n(t) \end{cases} \tag{1}$$

where $p(t)$ is the pilot tone signal, and $n(t)$ is stationary additive white Gaussian noise (AWGN) with power spectral density $N_0/2$. The pilot tone signal can be written as:

$$p(t) = \sqrt{2P}\, a(t-\tau)\cos[2\pi(f_0 + \Delta f)t + \theta] \tag{2}$$

where P is the pilot power, f_0 is the nominal carrier frequency, τ is the delay, Δf is the unknown frequency offset, and θ is the unknown carrier phase. Finally, $a(t)$ is the real PN sequence, which can be written as

$$a(t) = \sum_{k=-\infty}^{+\infty} a_k\, g(t - kT_c) \tag{3}$$

where $g(t)$ is the unit-energy chip pulse, and T_c is the transmission code chip period.

The received signal undergoes in-phase and quadrature (IQ) downconversion, Nyquist chip matched filtering and sampling, despreading through PN active correlation [2] as shown in fig.1. Despreading is normalized by a factor $1/\sqrt{M}$, where M is the coherent integration length. The IQ Gaussian random variables after despreading are therefore distributed with a variance equal to the input noise power spectral density.

Under H_1, the complex despread samples can be written as

$$\begin{aligned} x_k &= A(\Delta f)e^{j\theta}e^{j2\pi k\Delta f T} + n_k \\ &= s_k(\Delta f)e^{j\theta} + n_k \\ \text{for} \quad & k = 1, \ldots, L \end{aligned} \tag{4}$$

where L is the PDI length and

$$A(\Delta f) = \sqrt{PT}\,\mathrm{sinc}(\Delta f T) \tag{5}$$

$$s_k(\Delta f) = A(\Delta f)e^{j2\pi k\Delta f T} \tag{6}$$

and T is the coherent integration time (T=MT_c).

Observing the resulting complex symbol expression after despreading, it emerges that the presence of the frequency error produces an additional phase shift and a signal amplitude degradation quantified by the $\mathrm{sinc}(\Delta f T)$ term. This explains the necessity to limit appropriately the coherent integration length.

We introduce the vectorial notation

$$\bar{x} = [\ x_1 \quad \cdots \quad x_L\ ]^T \tag{7}$$

$$\bar{s}(\Delta f) = [\ s_1(\Delta f) \quad \cdots \quad s_L(\Delta f)\ ]^T \tag{8}$$

$$\bar{n} = [\ n_1 \quad \cdots \quad n_L\ ]^T \tag{9}$$

so that (4) can be rewritten as

$$\bar{x} = \bar{s}(\Delta f)e^{j\theta} + \bar{n} \tag{10}$$

which represents the vectorial form of the complex despread received symbols.

III. Likelihood Ratio Test

The probability density function (pdf) of $\bar{x}$ under H_1 and H_0, given the unknown Δf and θ, is easily shown to be

$$p(\bar{x}|H_1, \Delta f, \theta) = \frac{1}{(2\pi\sigma^2)^L}\exp\left\{-\frac{\|\bar{x} - \bar{s}(\Delta f)e^{j\theta}\|^2}{2\sigma^2}\right\} \tag{11}$$

$$p(\bar{x}|H_0, \Delta f, \theta) = \frac{1}{(2\pi\sigma^2)^L}\exp\left\{-\frac{\|\bar{x}\|^2}{2\sigma^2}\right\} \tag{12}$$

Therefore the likelihood ratio can be determined as a function of the unknown Δf and θ

$$\begin{aligned} \ell(\Delta f, \theta) &= \exp\left\{-\frac{LA^2(\Delta f)}{2\sigma^2}\right\} \cdot \\ &\cdot \exp\left\{\frac{\mathrm{Re}\{\bar{x}\cdot\bar{s}^*(\Delta f)e^{-j\theta}\}}{\sigma^2}\right\} \end{aligned} \tag{13}$$

To eliminate the dependence on the unknown phase, the theoretical approach based on the average likelihood ratio test is followed [8]. Assume θ is a random parameter with uniform pdf

$$p_\theta(\theta) = \frac{1}{2\pi} \quad , \quad \theta \in [-\pi, \pi[\tag{14}$$

As a consequence, the resulting average likelihood ratio (ALR) is dependent on the frequency error only. Then, by taking the logarithmic form of the obtained ALR, and assuming to operate at low SNR, the average log-likelihood ratio (ALLR), $\Lambda(\Delta f)$, can be found:

$$\Lambda(\Delta f) = \frac{A^2(\Delta f)}{4\sigma^4}\left\{|\sum_{k=1}^{L} x_k e^{-j2\pi k\Delta f T}|^2 - 2L\sigma^2\right\} \tag{15}$$

Note that if the frequency error tends to zero, then the ALLR reduces to coherent integration over the total PN period, LT, with a final IQ square and sum, i.e. the well-known solution with no PDI, which in this way is theoretically confirmed to be optimal if and only if there is no frequency error.

In the scenario under evaluation, a considerable frequency error can characterize the received signal. In this case, the design is carried out for the worst case, $\Delta f = \phi$, being ϕ the maximum foreseen frequency error value (which is usually known having fixed the particular system). To this aim, the coherent integration interval, T, is chosen so that $A(\phi)$ is sufficiently close to $A(0)$, in order to minimize losses. Operating in this way, the multiplicative constant becomes ineffective and can be incorporated in the ratio test threshold. So it follows

$$\Lambda(\Delta f) = |\sum_{k=1}^{L} x_k e^{-j2\pi k \Delta f T}|^2 \tag{16}$$

A. Frequency Error Estimation

The problem of eliminating the dependence on Δf out of the ALLR is firstly tackled through the maximum likelihood approach. Δf is modelled as a deterministic unknown parameter, and the related generalized likelihood ratio is

$$\Lambda = \max_{\Delta f} \Lambda(\Delta f) \tag{17}$$

As the the direct procedure of letting $\frac{d\Lambda(\Delta f)}{d\Delta f} = 0$ does not lead to any practical result, an appealing solution may be found by proceeding as in [9], [6], [7]. The likelihood ratio is written as the sum of L different terms, as

$$\Lambda(\Delta f) = \Lambda_0 + \sum_{n=1}^{L-1} \Lambda_n(\Delta f) \tag{18}$$

where

$$\Lambda_0 = \sum_{k=1}^{L} |x_k|^2 \tag{19}$$

is the classical NC-PDI, and

$$\Lambda_n(\Delta f) = 2\mathrm{Re}\left\{ \sum_{k=n+1}^{L} x_k x_{k-n}^* e^{-j2\pi n \Delta f T} \right\} \tag{20}$$

The generalized strategy is then applied on each single term, $\Lambda_n(\Delta f)$, as

$$\Lambda_n = \max_{\Delta f} \Lambda_n(\Delta f) \tag{21}$$

For this reason, the resulting scheme is identified as Single Term Maximum Likelihood-PDI (STML-PDI). The maximization procedure is repeated for all the L–1 terms dependent on the frequency error; thus, $L-1$ different estimated values for Δf are obtained, and not a single Δf estimate is provided by this technique. Nevertheless, an approximation of the maximum likelihood ratio test, which does not depend on the frequency offset value, can be derived in the form

$$\Lambda = \sum_{n=0}^{L-1} \Lambda_n \underset{\hat{H}_0}{\overset{\hat{H}_1}{\gtrless}} \xi \tag{22}$$

where Λ is the STML-PDI likelihood ratio, ξ is the test threshold, and $\hat{H}_i$, i=0,1 indicates the decided hypothesis. Λ_0 is the classic NC-PDI scheme, and Λ_1 corresponds to the D-PDI scheme. The additional $L-2$ terms

$$\Lambda_n = 2 |\sum_{k=n+1}^{L} x_k x_{k-n}^*| \tag{23}$$

are structured in a similar way with respect to the D-PDI, and are so identified as n-Span Differential terms [6], [7]. The larger is n, the more stringent the assumption of constant channel-induced phase rotation. Moreover, adding more terms yields a gain which becomes smaller and smaller, as verified by simulations [7]. Therefore, a practical upper limit to the sum (22) may be found. At least, it is possible to sum NC-PDI and D-PDI by considering

$$\Lambda(2) = \Lambda_0 + \Lambda_1 \tag{24}$$

and the corresponding STML[$\Lambda(2)$] system. Notice the weighting factor 2 applied to the D-PDI term in this summation.

B. Frequency Error Average

To solve the uncertainty with respect to the frequency error, the second novel approach is based on the averaging technique. This method assumes Δf to be a random uniformly distributed parameter described by the pdf

$$p_{\Delta f}(\Delta f) = \frac{1}{2\phi} \quad , \Delta f \in [-\phi, \phi[\tag{25}$$

where ϕ is the maximum frequency error value, a known quantity related to the particular system scenario. Averaging the ALLR (16) with respect to the frequency error, the likelihood ratio identified as $\mathrm{A^2LLR}$ is obtained, where the selected notation underlines the twofold averaging operation. It results

$$\Lambda = \Lambda_0 + \sum_{n=1}^{L-1} \mathrm{sinc}(2n\phi T)\ \Lambda_n \tag{26}$$

where, introducing a nomenclature similar to the previous subsection, Λ_0 is again the classical NC-PDI, and

$$\Lambda_n = 2 \sum_{k=n+1}^{L} \mathrm{Re}\{x_k x_{k-n}^*\} \tag{27}$$

is the n-Span Differential term which, differently from the maximum likelihood approach, applies the real part instead of the module. Further, each Λ_n term is summed weighted by a coefficient $\mathrm{sinc}(2n\phi T)$. The subsequent decision test, denoted as A^{2}LLRT, follows immediately.

In those scenarios where the maximum frequency error value, ϕ, is quite large respect to $1/2T$, the weighting coefficient, $\mathrm{sinc}(2n\phi T)$, for $n > 1$, is small and possibly negative. Therefore, it is possible to arrest the sum simply to NC-PDI. When $\phi << 1/2T$, the contribution coming from the further terms becomes significant. Again, and more strongly, adding more terms yields a gain which becomes smaller and smaller: it is therefore wise and possibly convenient to arrest opportunely the sum (26) before $n=L-1$. At least the bound obtained by arresting the sum at the second term can be considered, introducing the A^{2}LLRT[$\Lambda(2)$] scheme

$$\Lambda(2) = \Lambda_0 + \mathrm{sinc}(2\phi T)\ \Lambda_1 \tag{28}$$

IV. Numerical results

In the following, simulation results are reported for STML-PDI and A^{2}LLRT systems in the AWGN channel, to provide performance comparisons between the two proposed solutions. System performance is considered in terms of Receiver Operating Characteristics (ROC), i.e. missed detection probability versus false alarm probability, at different threshold values. The comparison is carried out as a function of the actual frequency error affecting the received signal, which is for generality considered normalized with respect to the coherent integration rate, introducing $\nu = \Delta f T$. The SNR is set equal to 2dB, and the PDI length is L=16.

First of all, the partial cases of STML[$\Lambda(2)$] and A^{2}LLRT[$\Lambda(2)$] are considered, which have been introduced as upper bounds for the proposed solutions. For the average approach, a maximum frequency error value, $\phi = 1/4T$, is considered, which provides a degradation coefficient for Λ_1 equal to $\mathrm{sinc}(2\phi T) = 0.637$, and corresponds to a maximum normalized shift equal to 0.25. In fig.2 the performance comparison between A^{2}LLRT[$\Lambda(2)$] and STML[$\Lambda(2)$] is shown for a normalized frequency error equal to $\nu = 0.1$. Also NC-PDI and D-PDI are reported as a reference. Notice that both the proposed methods outperform the classical approaches, and the average solution achieves the best performance at low missed detection probabilities. If the frequency error increases, the STML case becomes better than the average case, as reported in fig.3 where $\nu = 0.2$ is selected. Furthermore, the average solution degrades also with respect to classic procedures.

The explanation of this behavior comes from the observation that while the frequency error is moderately small, the lower noise enhancement introduced by the average approach through application of the real part instead of the module is the dominant effect. When instead the frequency error increases (above a certain context-dependent threshold, that we have not specifically dimensioned), the gain introduced by applying the non linearity after having accumulated over the samples in the STML case becomes significant. This fact suggests that the A^{2}LLRT[$\Lambda(2)$] solution can be profitably employed in those systems where the maximum frequency error can be considered not too large. Notice, however, that a normalized frequency error of 0.1 is a typical practical value. Finally, note that the STML case confirms to be the best solution [6], [7] also at large frequency offsets.

Fig.4 reports the performance of the total systems A^{2}LLRT and STML for $\nu = 0.1$. The total generalized solution outperforms its bound [6], [7]. Instead, the total average scheme is nearby the A^{2}LLRT[$\Lambda(2)$] case, highlighting that for relatively small frequency offsets no gain is introduced by summing further terms. The STML total case is therefore the best solution, introducing however the drawback of the increased associated complexity. Fig.5 shows the same comparison for a larger frequency error, i.e. ν=0.2. In this case, a performance improvement of the total average system with respect to its bound can be noticed, approaching the STML[$\Lambda(2)$] case. But the total STML solution provides again the best behavior.

Taking into account the complexity increase associated to the introduction of the STML[Λ] scheme in the receiver, the possibility to adopt a A^{2}LLRT[$\Lambda(2)$] solution to perform PDI in the presence of a relatively small frequency offset appears to be a profitable design choice, favorably solving the trade-off between performance and complexity.

V. Conclusions

In this paper theoretical PDI methods have been presented for code acquisition in the presence of a frequency error in the AWGN channel. The envisaged approaches are based on frequency error estimation (STML system) and average (A^2LLRT case), respectively. Performance comparison among the novel solutions and classic Non Coherent and Differential PDI schemes was conducted through simulations. Numerical results reveal that the STML solution outperforms traditional schemes for each operational condition in terms of frequency offset. The A^2LLRT approach outperforms other schemes in a limited typical range of frequency errors.

Acknowledgement

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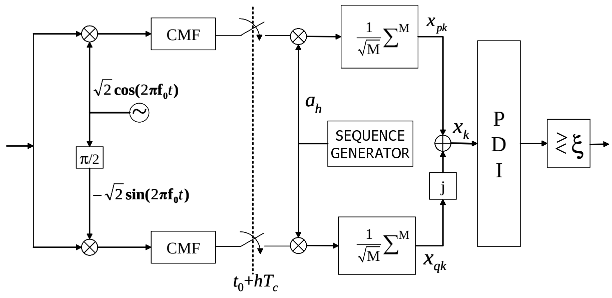

Fig. 1. System model

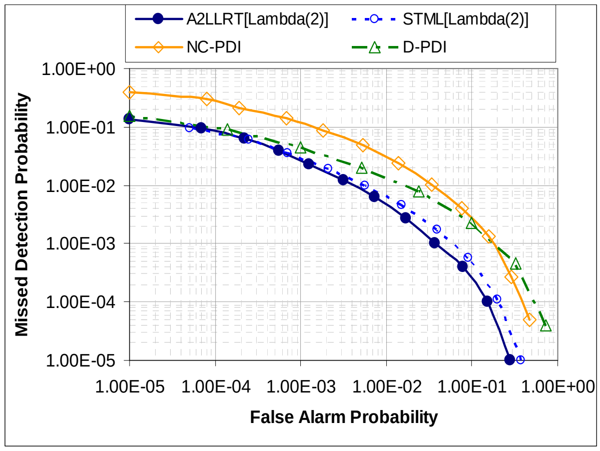

Fig. 2. Comparison among A^2LLRT[$\Lambda(2)$], STML[$\Lambda(2)$], NC-PDI, and D-PDI, SNR=2dB, L=16, $\nu = 0.1$

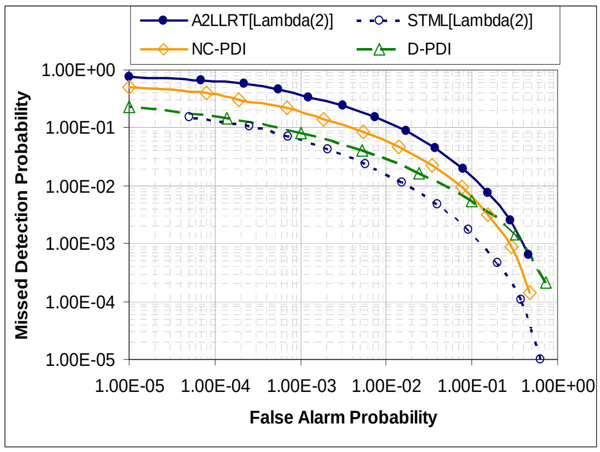

Fig. 3. Comparison among A^2LLRT[$\Lambda(2)$], STML[$\Lambda(2)$], NC-PDI, and D-PDI, SNR=2dB, L=16, $\nu = 0.2$

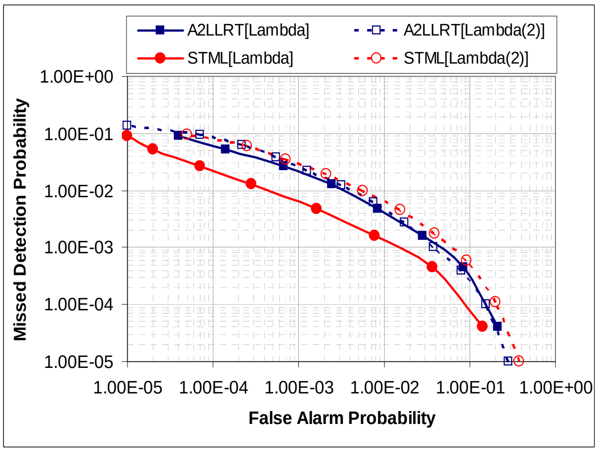

Fig. 4. Comparison among A^2LLRT[Λ], STML[Λ], A^2LLRT[$\Lambda(2)$], and STML[$\Lambda(2)$], SNR=2dB, L=16, $\nu = 0.1$

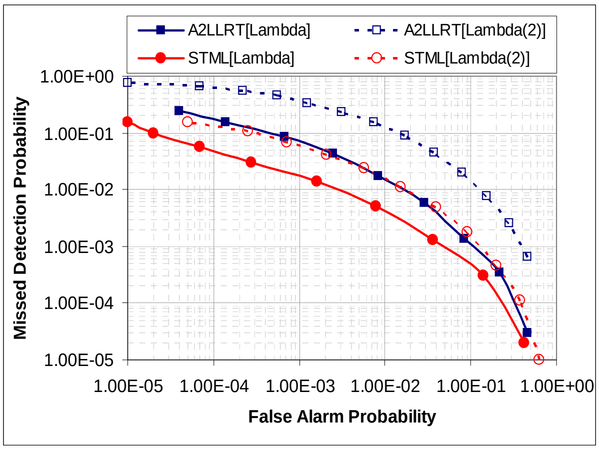

Fig. 5. Comparison among A^2LLRT[Λ], STML[Λ], A^2LLRT[$\Lambda(2)$], and STML[$\Lambda(2)$], SNR=2dB, L=16, $\nu = 0.2$